LÍMITES

Problema 19:

Calcular:

$$\lim_{n\to\infty} \sqrt{2n^2+3n-2}-\sqrt{2n^2+2}$$

Solución problema 19:

Comprobamos que es una indeterminación:

$$\lim_{n\to\infty} \sqrt{2n^2+3n-2}-\sqrt{2n^2+2} =$$

$$\sqrt{2\infty^2+3\cdot\infty-2}-\sqrt{2\infty^2+2} = \infty - \infty$$

Ahora, lo calculamos, para ello multiplicamos por el conjugado:

$$\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}$$

Así:

$$\lim_{n\to\infty} \sqrt{2n^2+3n-2}-\sqrt{2n^2+2}\cdot\frac{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}} =$$

$$\lim_{n\to\infty} \frac{\left(\sqrt{2n^2+3n-2}-\sqrt{2n^2+2}\right)\cdot\left(\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}\right)}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}} =$$

En el numerador tenemos una diferencia por suma que es igual a diferencia de cuadrados:

$$\lim_{n\to\infty}\frac{\sqrt{2n^2+3n-2}^2-\sqrt{2n^2+2}^2}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}}=\lim_{n\to\infty}\frac{2n^2+3n-2-(2n^2+2)}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}}=\lim_{n\to\infty}\frac{\cancel{2n^2}+3n-2-\cancel{2n^2}-2}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}}=$$

$$=\lim_{n\to\infty}\frac{3n-4}{\sqrt{2n^2+3n-2}+\sqrt{2n^2+2}}=\lim_{n\to\infty}\frac{\frac{3n}{n}-\frac{4}{n}}{\sqrt{\frac{2n^2}{n^2}+\frac{3n}{n^2}-\frac{2}{n^2}}+\sqrt{\frac{2n^2}{n^2}+\frac{2}{n^2}}}=\lim_{n\to\infty}\frac{3-\frac{4}{n}}{\sqrt{2+\frac{3}{n}-\frac{2}{n^2}}+\sqrt{2+\frac{2}{n^2}}}=$$

$$\frac{3}{\sqrt{2+\frac{3}{\infty}-\frac{2}{\infty^2}}+\sqrt{1+\frac{2}{\infty^2}}}=\frac{3}{\sqrt{2+0+0}+\sqrt{2+0}}=\frac{3}{\sqrt{2}+\sqrt{2}}=\frac{3}{2\sqrt{2}}=\frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2\cdot 2}=\mathbf{\frac{3\sqrt{2}}{4}}$$