PROBLEMAS DE EXPRESIONES ALGEBRÁICAS Y OPERACIONES

PROBLEMA 130:

Resolver:

$$\frac{3+\sqrt{6}}{\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)}$$

Solución Problema 130:

$$\frac{3+\sqrt{6}}{\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)}$$

El conjugado de:

$$\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)$$

Es:

$$\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)$$

Luego:

$$\frac{3+\sqrt{6}}{\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)}\cdot\frac{\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)}{\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)}$$

Ahora,

$$\left[\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)\right]\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]$$

Es una identidad notable: suma por diferencia o diferencia por suma (como en nuestro ejemplo) igual a diferencia de cuadrados:

$$\left[\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)\right]\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]=\left(5\sqrt{3}-2\sqrt{12}\right)^2-\left(\sqrt{32}-\sqrt{50}\right)^2$$

A continuación, lo calculamos:

$$\left(5\sqrt{3}-2\sqrt{12}\right)^2-\left(\sqrt{32}-\sqrt{50}\right)^2=\left(5^2\cdot\sqrt{3}^2+2^2\cdot\sqrt{12}^2-2\cdot 5\sqrt{3}\cdot 2\sqrt{12}\right)-\left(\sqrt{32}^2+\sqrt{50}^2-2\cdot\sqrt{32}\cdot\sqrt{50}\right)=$$

$$=\left(25\cdot 3+4\cdot 12-20\sqrt{36}\right)-\left(32+50-2\cdot\sqrt{32}\cdot\sqrt{50}\right)=(75+48-20\cdot 6)-\left(82-2\cdot\sqrt{32\cdot 50}\right)=$$

$$=(75+48-20\cdot 6)-\left(82-2\cdot\sqrt{32\cdot 50}\right)=(123-120)-\left(82-2\cdot\sqrt{1600}\right)=3-(82-2\cdot 40)=$$

$$=3-(82-80)=3-2=1$$

Por tanto, el denominador de la fracción es igual a 1:

$$\left[\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)\right]\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]=1$$

Sustituimos su valor en la identidad inicial:

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$$\frac{3+\sqrt{6}}{\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)}\cdot\frac{\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)}{\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)}=\frac{\left(3+\sqrt{6}\right)\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]}{\left[\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)\right]\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]}$$

$$=\frac{\left(3+\sqrt{6}\right)\cdot\left[\left(5\sqrt{3}-2\sqrt{12}\right)+\left(\sqrt{32}-\sqrt{50}\right)\right]}{1}=\left(3+\sqrt{6}\right)\cdot\left[\left(5\sqrt{3}-2\sqrt{3\cdot 2^2}\right)+\left(\sqrt{2^5}-\sqrt{2\cdot 5^2}\right)\right]=$$

$$=\left(3+\sqrt{6}\right)\cdot\left[\left(5\sqrt{3}-4\sqrt{3}\right)+\left(4\sqrt{2}-5\sqrt{2}\right)\right]=\left(3+\sqrt{6}\right)\cdot\left[\left(\sqrt{3}\right)+\left(-\sqrt{2}\right)\right]=\left(3+\sqrt{6}\right)\cdot\left[\sqrt{3}-\sqrt{2}\right]=$$

$$=3\sqrt{3}-3\sqrt{2}+\sqrt{18}-\sqrt{12}=3\sqrt{3}-3\sqrt{2}+\sqrt{2\cdot 3^2}-\sqrt{3\cdot 2^2}=3\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{3}=3\sqrt{3}-2\sqrt{3}-3\sqrt{2}+3\sqrt{2}=\sqrt{3}$$

Luego,

$$\frac{3+\sqrt{6}}{\left(5\sqrt{3}-2\sqrt{12}\right)-\left(\sqrt{32}-\sqrt{50}\right)}=\sqrt{3}$$